

Quintessence Cosmology with an Effective Λ -Term in Lyra Manifold

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Abstract

In this paper, we study quintessence cosmology with an effective Λ -term in Lyra manifold. We consider three different models by choosing variable Λ depend on time, the Hubble parameter and the energy density of dark matter and dark energy. Dark energy assumed as quintessence which interacts with the dark matter. By using numerical analysis we investigate behavior of cosmological parameters in three different models and compare our results with observational data. Statefinder diagnostic is also performed for all models.

1 Introduction

Accelerated expansion of universe can be described by dark energy which has positive energy and negative pressure [1, 2]. There are several theories to describe the dark energy such as Einstein's cosmological constant which has two crucial problems so called fine tuning and coincidence [3]. There are also other interesting models to describe the dark energy such as k -essence model [4], tachyonic models [5] and Chaplygin gas models [6-20]. An interesting model to describe dark energy is called quintessence [21-23]. Quintessence is described by a canonical scalar field ϕ minimally coupled to gravity. Compared to other scalar-field models such as phantoms and k -essence, quintessence is the simplest scalar-field scenario without having theoretical problems such as the appearance of ghosts and Laplacian instabilities.

On the other hand, The Lyra's geometry provides one of the possible alternatives in modification of the cosmological models [24]. Such a modification of the gravitational theory has long been known, but now it again attracts attention due to the opening of the late-time cosmological acceleration. The effective cosmological term in Lyra's geometry recently has been studied by the several works, such as [25, 26].

Moreover, it is well known that Einstein equations of general relativity do not permit any variations in the gravitational constant G and cosmological constant Λ because of the fact that the Einstein tensor has zero divergence and energy conservation law is also zero. So, some modifications of Einstein equations are necessary. Therefore, the study of the varying G and Λ can be done only through modified field equations and modified conservation laws. Already we construct several cosmological models based on variation of G and Λ [27-31].

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Finally we should say about importance of interactions in cosmological models. One of the ways to solve the cosmological coincidence problem is to consider the interaction between the components on phenomenological level. Also by consideration of interaction between dark matter and dark energy we can construct a real model of universe [32-35].

This paper is organized as the follows. In section 2 we review quintessence cosmology, and in section 3 we introduce our models. In section 4 we write field equations which should solved to obtain behavior of cosmological parameters. In section 5 we consider special case of constant G and Λ . In section 6 we give results of our numerical analysis about cosmological parameters in three different models. In section 7 statefinder diagnostic will perform and numerically will analyze. Finally, in section 8 we give conclusions.

2 Quintessence cosmology

Quintessence is a scalar field model for the dark energy described by the field ϕ and the potential $V(\phi)$. Energy density and pressure are given as the follow,

$$\rho_Q = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad (1)$$

and,

$$\rho_b = \frac{1}{2}\dot{\phi}^2 - V(\phi), \quad (2)$$

where ρ_Q denotes the density of dark energy and ρ_b denotes the density of a barotropic fluid which will described dark matter in universe with the equation of state of the form $P_b = \omega_b \rho_b$. We consider a model for the universe where an effective energy density and pressure assumed to be given as the follows,

$$\rho = \rho_Q + \rho_b, \quad (3)$$

and,

$$P = P_Q + P_b. \quad (4)$$

3 The models

First of all we introduce an interaction Q between dark energy and dark matter as follow,

$$Q = 3Hb(\rho_b + \rho_Q), \quad (5)$$

where b is a positive constant. The solving strategy and structure of the problem is the following that we will assume that the form of the potential $V(\phi)$ is given,

$$V(\phi) = V_0 e^{[-\frac{\alpha}{2}\phi^\gamma]}, \quad (6)$$

where α and γ are arbitrary constants. Moreover, we will consider 3 different forms of Λ as the follows. In the first model we assume that,

$$\Lambda = \rho_Q + \rho_b e^{[-tH]}. \quad (7)$$

It is clear that at the late time, where the density of dark energy is infinitesimal constant, the value of Λ also takes infinitesimal constant, which may be agree with observational data.

In the second model we assume that,

$$\Lambda = H^2 + (\rho_b + \rho_Q)e^{[-tH]}. \quad (8)$$

Again, after late time, the value of Λ takes the value H_0^2 . Finally in the third model we consider the following relation,

$$\Lambda = t^{-2} + \rho_Q + \rho_b e^{[-tH]}. \quad (9)$$

The late time behavior of this model is similar to the first model.

According to the relations (7), (8) and (9) we have three different models which will analyze in the next sections.

4 The field equations

Field equations that govern our model of consideration are,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \Lambda g_{\mu\nu} + \frac{3}{2}\phi_\mu\phi_\nu - \frac{3}{4}g_{\mu\nu}\phi^\alpha\phi_\alpha = T_{\mu\nu}. \quad (10)$$

Considering the content of the universe to be a perfect fluid, we have,

$$T_{\mu\nu} = (\rho + P)u_\mu u_\nu - P g_{\mu\nu}, \quad (11)$$

where $u_\mu = (1, 0, 0, 0)$ is a 4-velocity of the co-moving observer, satisfying $u_\mu u^\mu = 1$. Let ϕ_μ be a time-like vector field of displacement,

$$\phi_\mu = \left(\frac{2}{\sqrt{3}}\beta, 0, 0, 0 \right), \quad (12)$$

where $\beta = \beta(t)$ is a function of time alone, and the factor $\frac{2}{\sqrt{3}}$ is substituted in order to simplify the writing of all the following equations. By using FRW metric for a flat universe,

$$ds^2 = -dt^2 + a(t)^2 (dr^2 + r^2 d\Omega^2), \quad (13)$$

the field equations can be reduced to the following Friedmann equations,

$$3H^2 - \beta^2 = \rho + \Lambda, \quad (14)$$

and,

$$2\dot{H} + 3H^2 + \beta^2 = -P + \Lambda, \quad (15)$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter, and dot stands for differentiation with respect to cosmic time t , $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$, and $a(t)$ represents the scale factor. The θ and ϕ parameters are the usual azimuthal and polar angles of spherical coordinates, with $0 \leq \theta \leq \pi$ and $0 \leq \phi < 2\pi$.

The continuity equation reads as,

$$\dot{\rho} + \dot{\Lambda} + 2\beta\dot{\beta} + 3H(\rho + P + 2\beta^2) = 0. \quad (16)$$

If we assume that,

$$\dot{\rho} + 3H(\rho + P) = 0, \quad (17)$$

then, Eq. (16) will give a link between Λ and β of the following form,

$$\dot{\Lambda} + 2\beta\dot{\beta} + 6H\beta^2 = 0. \quad (18)$$

To introduce an interaction between the dark energy and dark matter, we should mathematically split Eq. (17) into two following equations,

$$\dot{\rho}_b + 3H(\rho_b + P_b) = Q, \quad (19)$$

and,

$$\dot{\rho}_Q + 3H(\rho_Q + P_Q) = -Q. \quad (20)$$

For the barotropic fluid with $P_b = \omega_b \rho_b$, equation (19) will take the following form,

$$\dot{\rho}_b + 3H(1 + \omega_b - b)\rho_b = 3Hb\rho_Q. \quad (21)$$

Cosmological parameters of our interest are EoS parameters of each fluid components $\omega_i = P_i/\rho_i$, EoS parameter of composed fluid,

$$\omega_{tot} = \frac{P_b + P_Q}{\rho_b + \rho_Q}, \quad (22)$$

deceleration parameter q , which can be written as,

$$q = \frac{1}{2} \left(1 + 3 \frac{P}{\rho} \right). \quad (23)$$

5 Case of constant G and Λ

Before analyzing our three models we assume the simplest case with constant G and Λ to obtain effect of varying Λ in the next section. According to this assumption, Eq.(16) will be modified as the following,

$$\dot{\rho} + 2\beta\dot{\beta} + 3H(\rho + P + 2\beta^2) = 0, \quad (24)$$

with $\dot{\rho} + 3H(\rho + P) = 0$, we have,

$$2\dot{\beta} + 6H\beta = 0. \quad (25)$$

Numerical analysis of the model gives the following behavior for the model.

From the Fig. 1 we can see that the Hubble expansion parameter is decreasing function of time and yields to a constant at the late time. It is illustrated that the interaction term increases value of the Hubble expansion parameter. Fig. 1 also contains deceleration parameter. We can see acceleration to deceleration phase transition. In the case of non-interacting case ($b = 0$) the final value of of deceleration parameter is positive which disagree with current observations. Therefore, presence of interaction is necessary to obtain $q \rightarrow -1$ or $q \geq -1$ in agreement with observational data.

Behavior of EoS parameters plotted in the Fig. 2. In this case also we find that presence of interaction term is necessary to obtain $\omega \rightarrow -1$. In the case of non-interacting component we see that $\omega \rightarrow 0$.

Finally in the plots of the Fig. 3 we see behavior of the scalar field and the potential. We see that scalar field is increasing function of time and decreased its value by increasing interaction strength. The situation for the potential is inverse. It is decreasing function of time which vanishes at the late time, and its value increased by increasing interaction strength.

We will compare our next results with the results of this section to obtain behavior of varying Λ in the models.

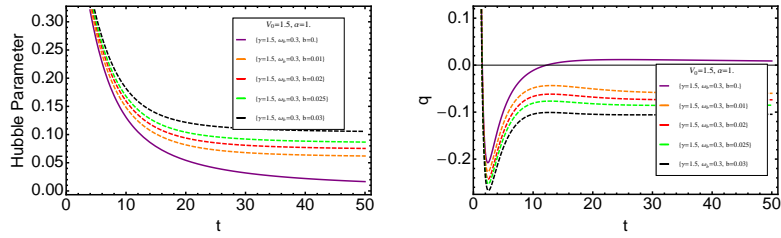


Figure 1: Behavior of Hubble parameter H and deceleration parameter q against t for constant G and Λ .

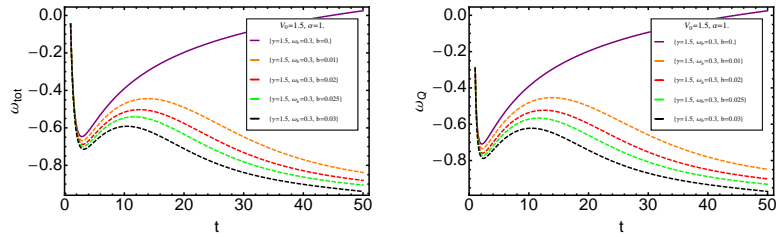


Figure 2: Behavior of EoS parameter ω_{tot} and ω_Q against t for constant G and Λ .

6 Numerical Results

In this section we study behavior of some cosmological parameters in three models by choosing different forms of Λ .

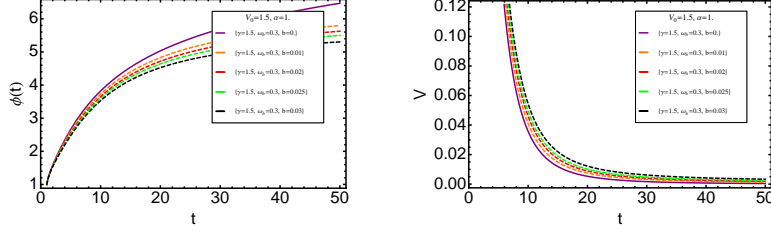


Figure 3: Behavior of filed ϕ and potential V against t for constant G and Λ .

6.1 Model 1

To describe the dynamics of the universe, we assume that the form of Λ is given by Eq. (7). The dynamics of $\beta(t)$ can be obtained from Eq. (18) as follow,

$$2\beta\dot{\beta} + 6H\beta^2 + \dot{\rho}_Q + \dot{\rho}_b - (H + t\dot{H})\rho_b e^{[-tH]} = 0. \quad (26)$$

Our numerical analysis yields to the following results. Plots of the Fig. 4 show behavior of the the Hubble expansion and the deceleration parameters with time. We can see that the the Hubble expansion parameter is decreasing function of time which yields to a constant at the late time. Comparing with the Fig. 1 suggests that consideration of varying Λ of the form given by the Eq. (7) decreases value of the Hubble expansion parameter. Also, behavior of the deceleration parameter agree with observational data ($0 \geq q \geq -1$) more than the case without interaction. Acceleration to deceleration phase transition also illustrated in the Fig. 4.

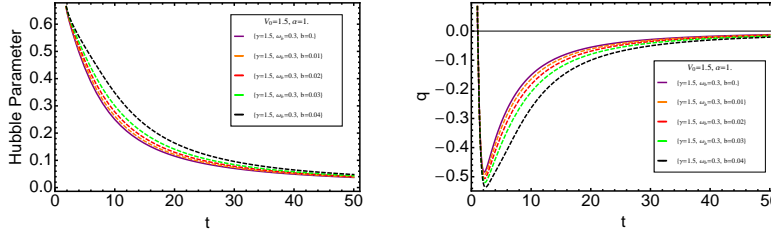


Figure 4: Behavior of Hubble parameter H and deceleration parameter q against t for Model 1.

Plots of the Fig. 5 show that value of EoS parameters are within $-1/3 \geq \omega \geq -1$. Their value decreased suddenly at the initial time and then grow to reach constant value. This is also agree with observational data.

In the Fig. 6 we draw the scalar field ϕ and the potential V . As expected, the potential vanished at the late time and the scalar field increases by time.

We can also obtain the late time behavior of $\beta(t)$ as the following,

$$\beta = Ce^{-3Ht}, \quad (27)$$

where C is an integration constant. In that case we can obtain $C_s^2 = 1$, where C_s is sound speed. Therefore, we find that the first model is stable, at least at the late time.

Our numerical analysis suggest the following densities,

$$\rho_b = \left(C - \int \frac{3C^2 e^{-6(1+H_0 t) - g(t)}}{t(1 + e^{-(1+H_0 t)})} dt \right) e^{g(t)}, \quad (28)$$

where constant H_0 is current value of the Hubble expansion parameter and,

$$g(t) = \int \frac{(1 + H_0 t) (e^{-(1+H_0 t)} - 3(1 + \omega))}{t(1 + e^{-(1+H_0 t)})} dt, \quad (29)$$

and,

$$\rho_Q = \left(C - \frac{C^2 \left(\frac{3H_0}{2t} (3 - \omega) \right)^{-\frac{1+3\omega}{4}} e^{-6 - \frac{3}{4} H_0 (3 - \omega) t} W(t)}{3H_0 (3 - \omega) \left(\frac{1+\omega}{2} \right)} \right) t^{-\frac{3}{2}(1+\omega)} e^{-\frac{3}{2} H_0 (1+\omega) t}, \quad (30)$$

where,

$$W(t) \equiv \text{WhittakerM} \left(\frac{1+3\omega}{4}, \frac{3(1+\omega)}{4}, \frac{3}{2} (3 - \omega) H_0 t \right). \quad (31)$$

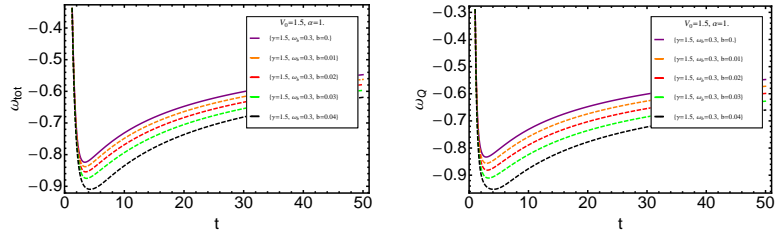


Figure 5: Behavior of EoS parameter ω_{tot} and ω_Q against t for Model 1.

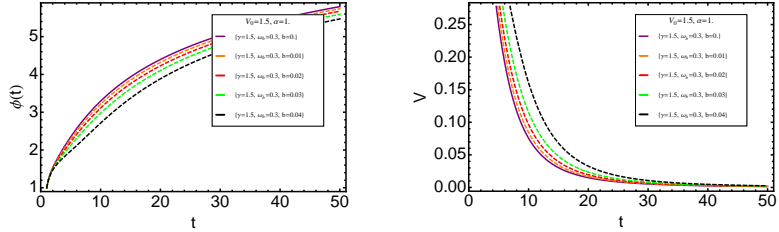


Figure 6: Behavior of field ϕ and potential V against t for Model 1.

6.2 Model 2

In the second model we use the relation (8) for Λ . The dynamics of $\beta(t)$ can be obtained from Eq. (18),

$$2\beta\dot{\beta} + 6H\beta^2 + 2H\dot{H} - ((H + t\dot{H})(\rho_Q + \rho_b) - \dot{\rho}_Q + \dot{\rho}_b)e^{[-tH]} = 0. \quad (32)$$

In the Fig. 7 we can see evolution of the Hubble expansion parameter (left) and deceleration parameter (right). It is clear that the Hubble parameter is decreasing function of time and yields to a constant value. This is similar to the previous model and lower than the the case without interaction. But, we can see differen behavior of deceleration parameter with previous model. It's value yields to zero at the late time, therefore we can't see agreement with current observational data. Therefore, we left this model and omit presentation of other parameters such as the EoS, the scalar field and the potential, and analyze the next model. However we can obtain behavior of these parameter similar to the previous model.

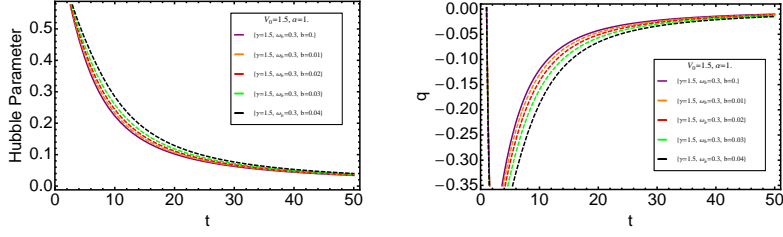


Figure 7: Behavior of Hubble parameter H and deceleration parameter q against t for Model 2.

6.3 Model 3

In the third model we use special form of Λ which is given by Eq. (9). Similar to the previous models we can draw plots of H and q (see Fig. 8). We can see similar behavior of the Hubble expansion parameter as previous models but evolution of the deceleration parameter don't show acceleration to deceleration phase transition. Although the value of q obtained within observational data but absence of mentioned transition yields us to left this model like the model 2.

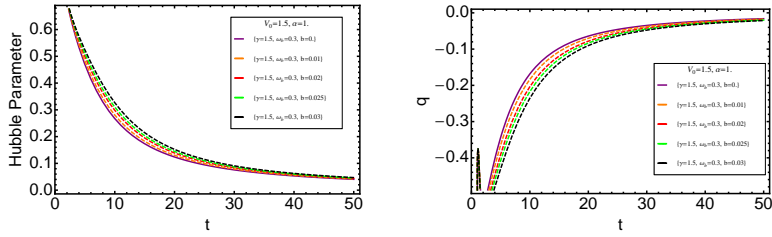


Figure 8: Behavior of Hubble parameter H and deceleration parameter q against t for Model 3.

7 Statefinder diagnostic

In the framework of general relativity, one of the properties of dark energy is that it is model-dependent and, in order to choose the best model of dark energy, a sensitive diagnostic tool is needed. The Hubble parameter H and the deceleration parameter q are very important quantities which can describe the geometric properties of the universe. Since $\dot{a} > 0$, hence $H > 0$ implies an expansion of the Universe. Moreover, $\ddot{a} > 0$, which means $q < 0$, indicates an accelerated expansion of the universe. Therefore, various dark energy models give $H > 0$ and $q < 0$, then they can not provide enough evidence to differentiate the more accurate cosmological observational data and the more general models of dark energy. For this aim, we need higher order of time derivative of scale factor and geometrical tool. Sahni *et.al* [36] proposed geometrical statefinder diagnostic tool, based on dimensionless parameters (r, s) which are function of scale factor and its higher order time derivatives. These parameters are defined as follow,

$$r = \frac{1}{H^3} \frac{\ddot{a}}{a} \quad s = \frac{r-1}{3(q-\frac{1}{2})}. \quad (33)$$

Results of our numerical analysis are presented in the Fig. 9. We can see that the first model has more agreement with observations [37, 38].

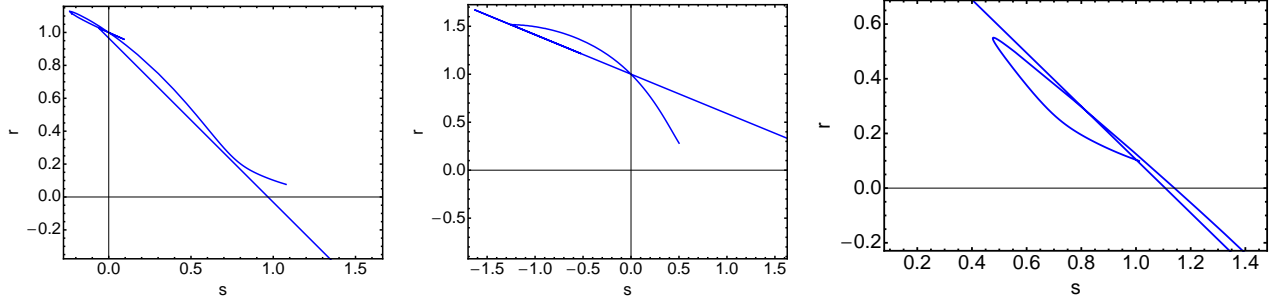


Figure 9: $r - s$: Model 1 (left), Model 2 (middle), Model 3 (right) .

8 Discussion

In this work we considered quintessence cosmology with an effective Λ -term in Lyra manifold which contains interaction between dark matter and dark energy. We assumed three different forms of variable Λ . In the first model the variable Λ is depend on dark energy density plus dark matter density multiple with exponential function of $(-Ht)$. The second model has variable Λ equal to squared Hubble parameter plus total density multiple with exponential function of $(-Ht)$. Finally the variable Λ of the third model contains the variable Λ of the first model plus inverse of squared time. Therefore late time behavior of the third model expected similar to the first model. Our numerical results show this agreement but acceleration to deceleration phase transition at the early time can't obtained in the third model. Also behavior of the deceleration parameter in the second model is disagree with the observational data. Therefore we conclude that the first model is the best model of this paper which agree with observational data. This point also verified with the statefinder diagnostic tool. We found that the first model is stable at the late time. Comparing our models with the case of constant Λ suggests that the first model with varying Λ yields to results which are in agreement with observational data more than the case of constant Λ . Also we found late time behavior of scalar field and obtained dark energy and dark matter densities of the first model by fitting plots.

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